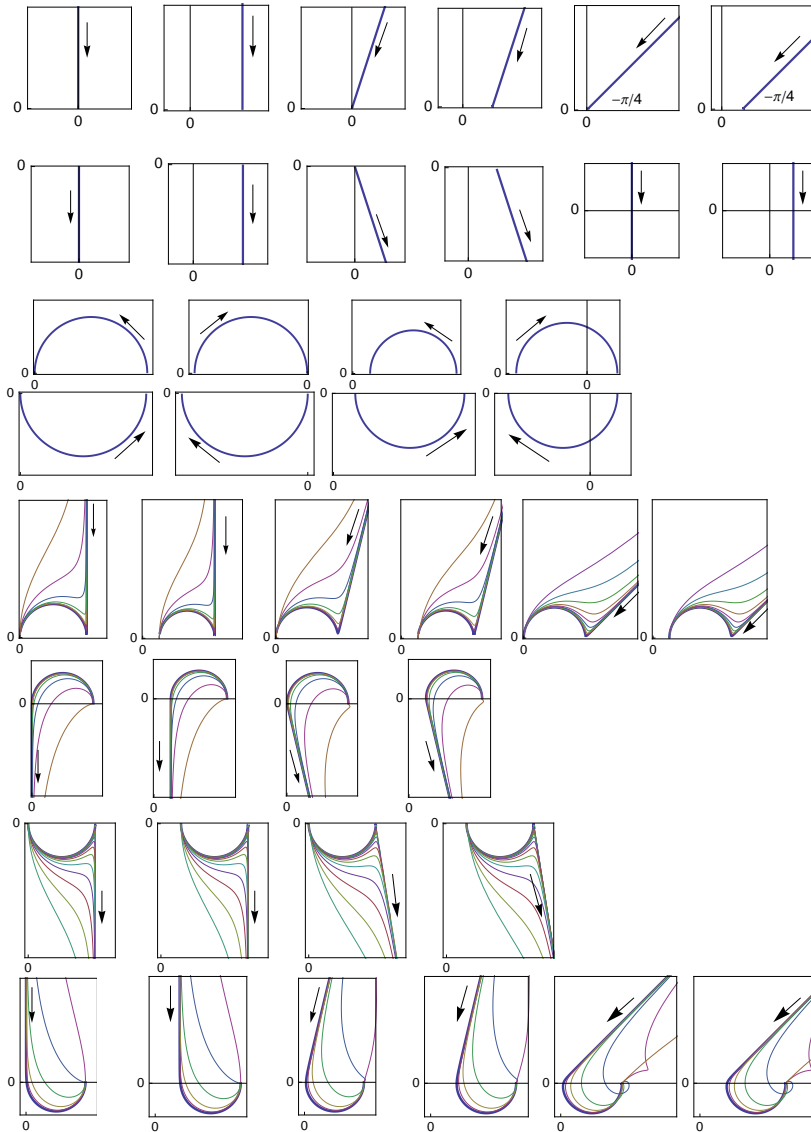


**Atlas**  
**of Electrochemical Nyquist Diagrams.**  
**I. One semicircle and straight line**  
A guide to select equivalent circuits



Bio-Logic @ [www.bio-logic.info](http://www.bio-logic.info)



September 28, 2011



# Contents

<b>1</b>	<b>Nyquist diagrams made of one straight line</b>	<b>5</b>
1.1	Capacitive straight line . . . . .	5
1.1.1	C1 element . . . . .	5
1.1.2	R1+C1 circuit . . . . .	6
1.1.3	Q1 element . . . . .	6
1.1.4	R1+Q1 circuit . . . . .	6
1.1.5	Q1 element, $\alpha_1 = 1/2$ . . . . .	6
1.1.6	W1 element . . . . .	6
1.1.7	R1+Q1 circuit, $\alpha_1 = 1/2$ . . . . .	6
1.1.8	R1+W1 circuit . . . . .	7
1.2	Inductive straight line . . . . .	7
1.2.1	L1 element . . . . .	7
1.2.2	R1+L1 circuit . . . . .	8
1.2.3	Q1 element, $\alpha_1 < 0$ . . . . .	8
1.2.4	R1+Q1 element, $\alpha_1 < 0$ . . . . .	8
1.2.5	L1+C1 circuit . . . . .	8
1.2.6	R1+L1+C1 circuit . . . . .	8
<b>2</b>	<b>Nyquist diagrams made of one semicircle</b>	<b>9</b>
2.1	One capacitive semicircle . . . . .	9
2.1.1	R1/C1 circuit . . . . .	9
2.1.2	R1+R2/C2 circuit . . . . .	10
2.1.3	R1/(R2+C2) circuit . . . . .	12
2.2	One inductive semicircle . . . . .	13
2.2.1	R1/L1 circuit . . . . .	13
2.2.2	R1/L1 circuit, $R_1 < 0$ . . . . .	13
2.2.3	R1+R2/L2 circuit . . . . .	14
2.2.4	R1+R2/L2 circuit, $R_2 < 0$ . . . . .	14
<b>3</b>	<b>Nyquist diagrams made of one semicircle and straight line</b>	<b>15</b>
3.1	Capacitive semicircle and straight line . . . . .	15
3.1.1	C1/(C2+R2) circuit . . . . .	15
3.1.2	C1+R2/C2 circuit . . . . .	15
3.1.3	R1+C1/(C2+R2) circuit . . . . .	16
3.1.4	R1+C1+R2/C2 circuit . . . . .	17
3.1.5	C1/(Q2+R2) circuit . . . . .	17
3.1.6	Q1+R2/C2 circuit . . . . .	17
3.1.7	R1+C1/(Q2+R2) circuit . . . . .	17

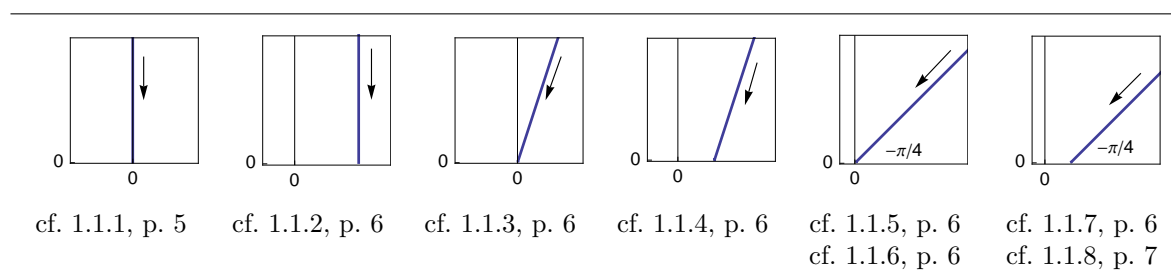
3.1.8	$R1+Q1+R2/C2$ circuit . . . . .	17
3.1.9	$C1/(R2+W2)$ circuit . . . . .	18
3.1.10	$R1+C1/(R2+W2)$ circuit . . . . .	18
3.1.11	$W1+R2/C2$ circuit . . . . .	18
3.1.12	$R1+W1+R2/C2$ circuit . . . . .	19
3.2	Capacitive semicircle and inductive straight line . . . . .	21
3.2.1	$L1+R2/C2$ circuit . . . . .	21
3.2.2	$R1+L1+R2/C2$ circuit . . . . .	21
3.2.3	$Q1+R2/C2$ circuit, $\alpha_1 < 0$ . . . . .	21
3.2.4	$R1+Q1+R2/C2$ circuit, $\alpha_1 < 0$ . . . . .	22
3.3	Inductive semicircle and straight line . . . . .	24
3.3.1	$R1/L1+L2$ circuit . . . . .	24
3.3.2	$L1/(R1+L2)$ circuit . . . . .	24
3.3.3	$R1+R2/L2+L3$ circuit . . . . .	24
3.3.4	$R1+L2/(R2+L3)$ circuit . . . . .	25
3.3.5	$R1/L1+Q1$ circuit, $\alpha_1 < 0$ . . . . .	25
3.3.6	$R1+R2/L2+Q1$ circuit, $\alpha_1 < 0$ . . . . .	25
3.4	Inductive semicircle and capacitive straight line . . . . .	27
3.4.1	$R1/L1+C1$ circuit . . . . .	27
3.4.2	$R1+R2/L2+C1$ circuit . . . . .	27
3.4.3	$R1/L1+Q1$ circuit . . . . .	27
3.4.4	$R1+R2/L2+Q1$ circuit . . . . .	27
3.4.5	$R1/L1+W1$ circuit . . . . .	28
3.4.6	$R1+R2/L2+W1$ circuit . . . . .	28

# Chapter 1

## Nyquist diagrams made of one straight line

### 1.1 Capacitive straight line

Table 1.1: Six different Nyquist diagrams made of one capacitive straight line.



(<sup>1</sup>)

#### 1.1.1 C1 element

Fig. 1.1.

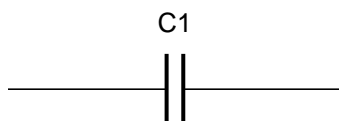


Figure 1.1: C1 element. For an electrochemical system C1 denotes the electrochemical double layer capacity  $C_{dl}$ .

<sup>1</sup> Nyquist diagrams are always plotted using convention of electrochemists: orthonormal parametric plot  $-\text{Im } Z$  vs.  $\text{Re } Z$ . The arrows always indicate the increasing frequency direction.

### 1.1.2 R1+C1 circuit

Fig. 1.2.

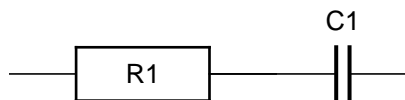


Figure 1.2: R1+C1 circuit. For an ideally polarized electrode C1 denotes the electrochemical double layer capacity  $C_{dl}$  and R1 the ohmic ( $R_{\Omega}$ ) or uncompensated resistance ( $R_u$ ) [1].

### 1.1.3 Q1 element

Fig. 1.3.

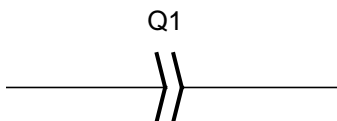


Figure 1.3: Q1 element (CPE element, cf. Handbook of Electrochemical Impedance Spectroscopy. Electrical circuits containing CPEs: <http://www.bio-logic.info/potentiostat/notesheis.html>)

### 1.1.4 R1+Q1 circuit

Fig. 1.4 [1, 2].

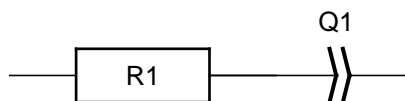


Figure 1.4: R1+Q1 circuit [1, 2].

### 1.1.5 Q1 element, $\alpha_1 = 1/2$

Fig. 1.3 with  $\alpha_1 = 1/2$ .

### 1.1.6 W1 element

Fig. 1.5.

### 1.1.7 R1+Q1 circuit, $\alpha_1 = 1/2$

Fig. 1.4 with  $\alpha_1 = 1/2$ .



Figure 1.5: W1 element (Warburg impedance, cf. Handbook of Electrochemical Impedance Spectroscopy. Diffusion impedances: <http://www.bio-logic.info/potentiostat/notesheis.html>).

### 1.1.8 R1+W1 circuit

Fig. 1.6.

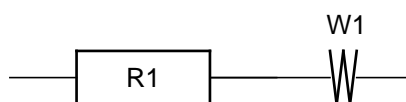
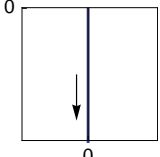
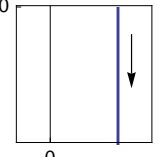
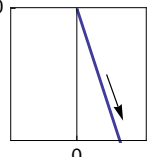
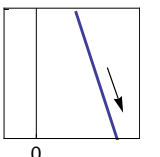
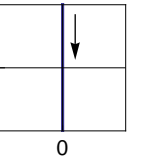
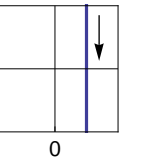


Figure 1.6: R1+W1 circuit.

## 1.2 Inductive straight line

Table 1.2: Four different Nyquist diagrams made of one inductive straight line and two diagrams made of one capacitive and one inductive straight line.

---

					
cf. 1.2.1, p. 7	cf. 1.2.2, p. 8	cf. 1.2.3, p. 8	cf. 1.2.4, p. 8	cf. 1.2.5, p. 8	cf. 1.2.6, p. 8

---

### 1.2.1 L1 element

Fig. 1.7.

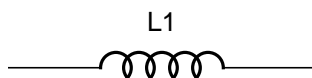


Figure 1.7: L1 element.

### 1.2.2 R1+L1 circuit

Fig. 1.8.

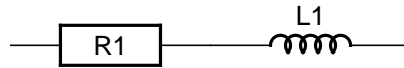


Figure 1.8: R1+L1 circuit.

### 1.2.3 Q1 element, $\alpha_1 < 0$

Fig. 1.3 with  $\alpha_1 < 0$ .

### 1.2.4 R1+Q1 element, $\alpha_1 < 0$

Fig. 1.4 with  $\alpha_1 < 0$ .

### 1.2.5 L1+C1 circuit

Fig. 1.9.

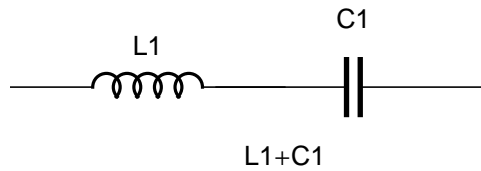


Figure 1.9: L1+C1 circuit.

### 1.2.6 R1+L1+C1 circuit

Fig. 1.10.

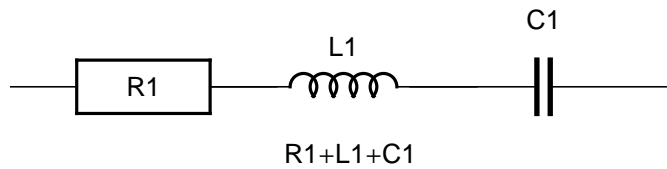


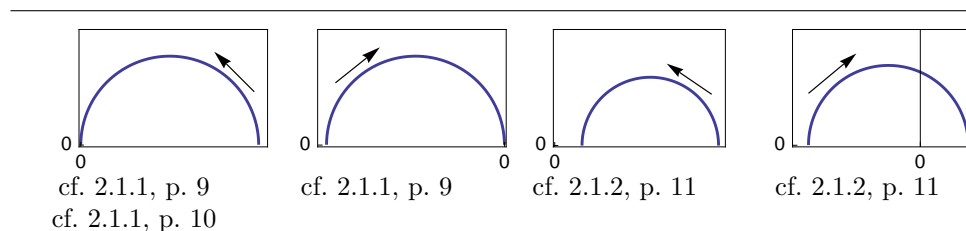
Figure 1.10: R1+L1+C1 circuit.

## Chapter 2

# Nyquist diagrams made of one semicircle

### 2.1 One capacitive semicircle

Table 2.1: Four different Nyquist diagrams made of one capacitive semicircle.



#### 2.1.1 R1/C1 circuit

Figs. 2.1, 2.2 <sup>(1)</sup>.

$R1, C1 > 0$

Fig. 2.2.

$R1 < 0, C1 > 0$

Fig. 2.2.

---

<sup>1</sup> Replacing C by Q (CPE element) in a parallel RC circuit changes a semicircle in a "depressed" semicircle arc, cf. Handbook of Electrochemical Impedance Spectroscopy. Electrical circuits containing CPEs: <http://www.bio-logic.info/potentiostat/notesheis.html>

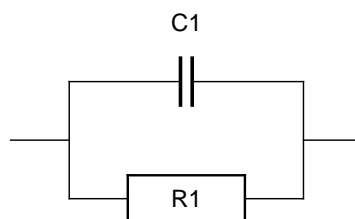


Figure 2.1: R1/C1 circuit. For an electrochemical system C1 denotes the electrochemical double layer capacity and R1 the charge transfer resistance  $R_{ct}$ .

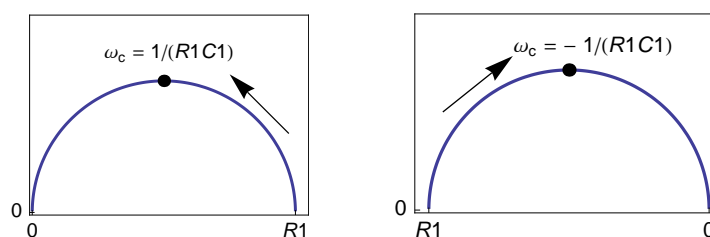


Figure 2.2: Nyquist diagrams of the impedance for the R1/C1 circuit. Left:  $R1, C1 > 0$ , right  $R1 < 0, C1 > 0$ .

### Voigt circuit with one time constant

Figs. 2.3, 2.4.

$$Z(\omega) = \sum_{i=1}^n \frac{Ri}{1 + i\omega\tau} = \frac{\sum_{i=1}^n Ri}{1 + i\omega\tau}, \quad \tau = RiCi$$

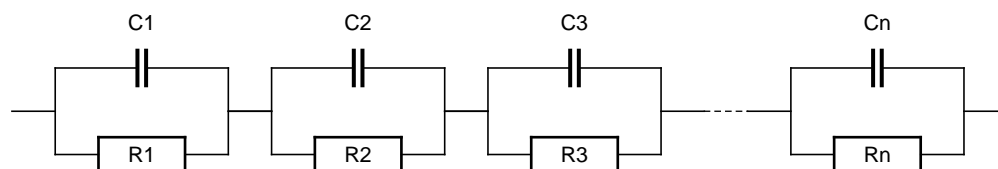


Figure 2.3: Voigt circuit.

### 2.1.2 R1+R2/C2 circuit

Figs. 2.5 <sup>(2)</sup>.

<sup>2</sup> The R1+R2/C2 and R1/(R2+C2) circuits are non-distinguishable, *i.e.* these circuits can be interchanged (cf. Handbook of Electrochemical Impedance Spectroscopy. Circuits made of resistors and capacitors: <http://www.bio-logic.info/potentiostat/notesheis.html>).

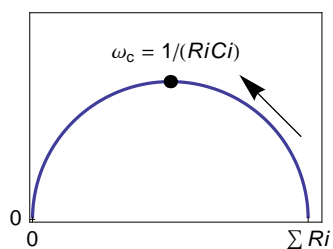


Figure 2.4: Nyquist diagram of the impedance for Voigt circuit with one time constant.

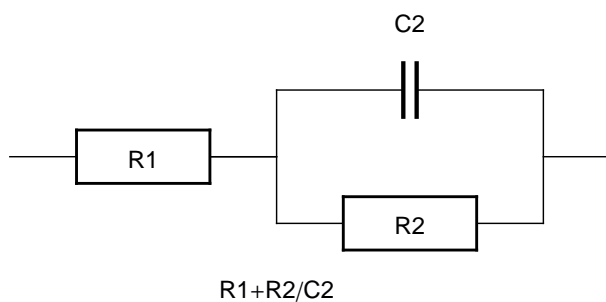


Figure 2.5:  $R1+R2/C2$  circuit. For an electrochemical system  $C2$  denotes the electrochemical double layer capacity  $C_{dl}$ ,  $R1$  the ohmic ( $R_{\Omega}$ ) or uncompensated resistance ( $R_u$ ) and  $R2$  the charge transfer resistance  $R_{ct}$ .

$R1, R2, C2 > 0$

Fig. 2.6.

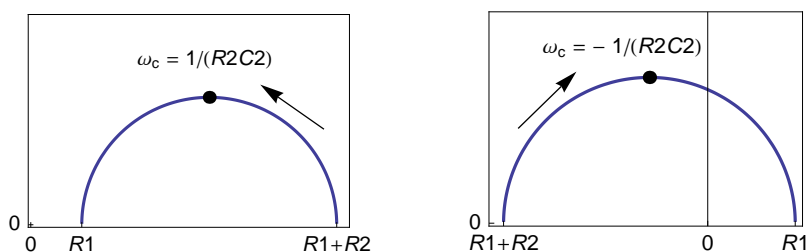


Figure 2.6: Nyquist impedance diagrams for the  $R1+R2/C2$  circuit. Left:  $R1, R2, C1 > 0$ , right:  $R1 < 0, R2 > 0, C1 > 0$ .

$R1 > 0, R2 < 0, C2 > 0$

Fig. 2.6.

### 2.1.3 $R1/(R2+C2)$ circuit

Figs. 2.7.

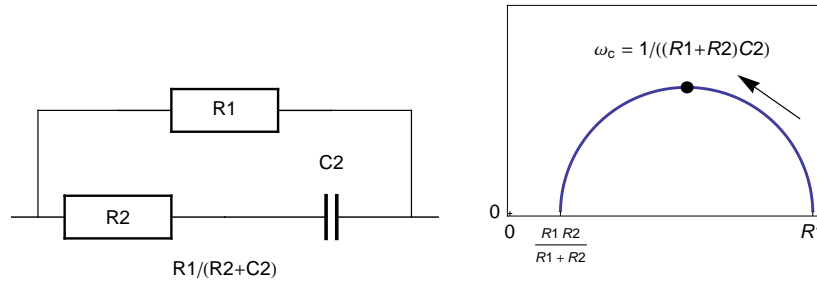
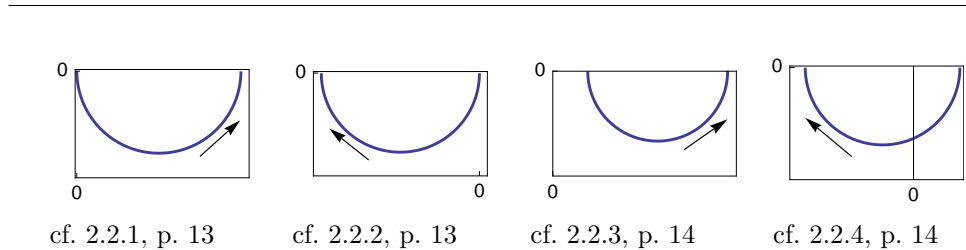


Figure 2.7:  $R1/(R2+C2)$  circuit and Nyquist impedance diagram.  $R1, R2, C1 > 0$

## 2.2 One inductive semicircle

Table 2.2: Four different Nyquist diagrams made of one inductive semicircle.



### 2.2.1 R1/L1 circuit

Figs. 2.8, 2.9.

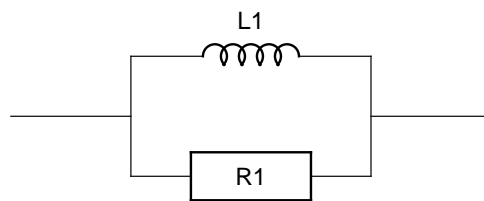


Figure 2.8: R1/L1 circuit.

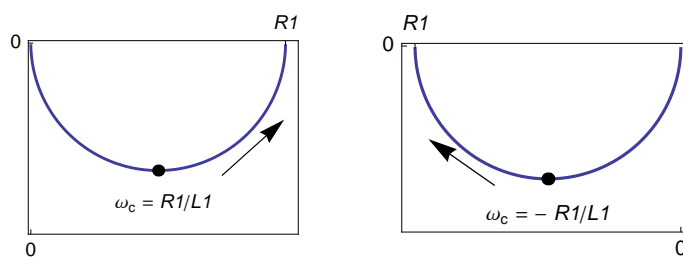


Figure 2.9: Nyquist impedance diagram for the R1/L1 circuit. Left:  $R1 > 0$ , right:  $R1 < 0$ .

### 2.2.2 R1/L1 circuit, $R1 < 0$

Fig. 2.8,  $R1 < 0$ .

### 2.2.3 $R1+R2/L2$ circuit

Figs. 2.10, 2.11.

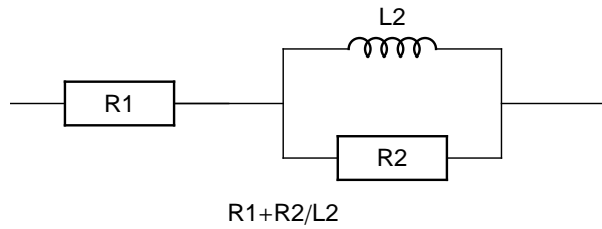


Figure 2.10:  $R1+ R2/L2$  circuit.

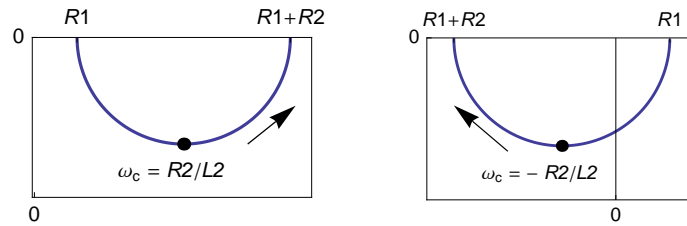


Figure 2.11: Nyquist impedance diagrams for the  $R1+R2/L2$  circuit. Left:  $R2 > 0$ , right:  $R2 < 0$ .

### 2.2.4 $R1+R2/L2$ circuit, $R2 < 0$

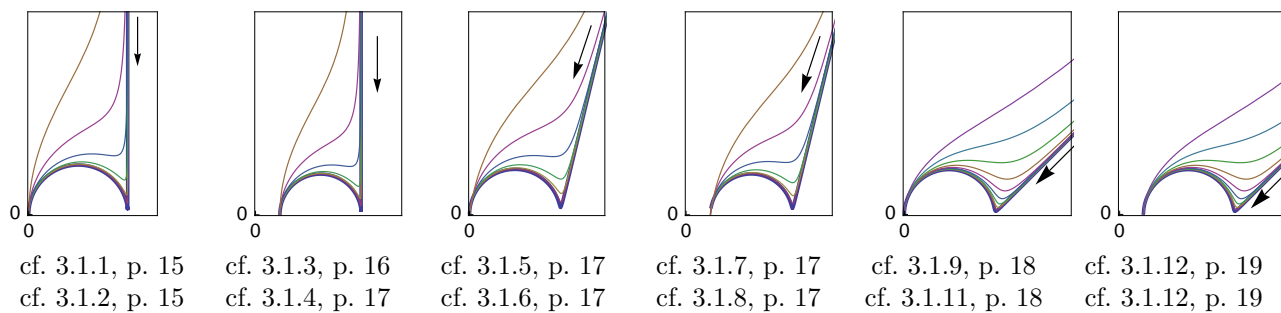
Figs. 2.10, 2.11.

## Chapter 3

# Nyquist diagrams made of one semicircle and one straight line

### 3.1 Capacitive semicircle and straight line

Table 3.1: Six different Nyquist diagrams made of one capacitive semicircle and one straight line (limiting cases).



#### 3.1.1 $C1/(C2+R2)$ circuit

Fig. 3.1 <sup>(1)</sup>.

#### 3.1.2 $C1+R2/C2$ circuit

Fig. 3.2.

<sup>1</sup>  $C1/(C2+R2)$  and  $C1+R2/C2$  circuits are non-distinguishable, *i.e.* these circuits can be interchanged (cf. Handbook of Electrochemical Impedance Spectroscopy. Circuits made of resistors and capacitors: <http://www.bio-logic.info/potentiostat/notesheis.html>).

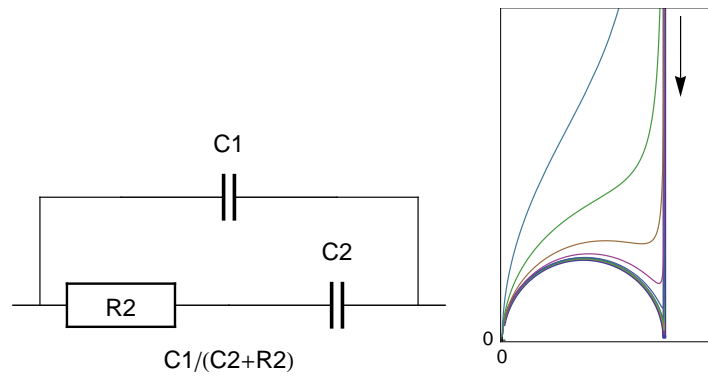


Figure 3.1:  $C_1/(C_2+R_2)$  circuit and Nyquist impedance diagrams.

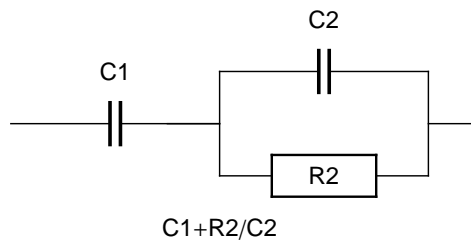


Figure 3.2:  $C_1+R_2/C_2$  circuit.

### 3.1.3 $R_1+C_1/(C_2+R_2)$ circuit

Fig. 3.3.

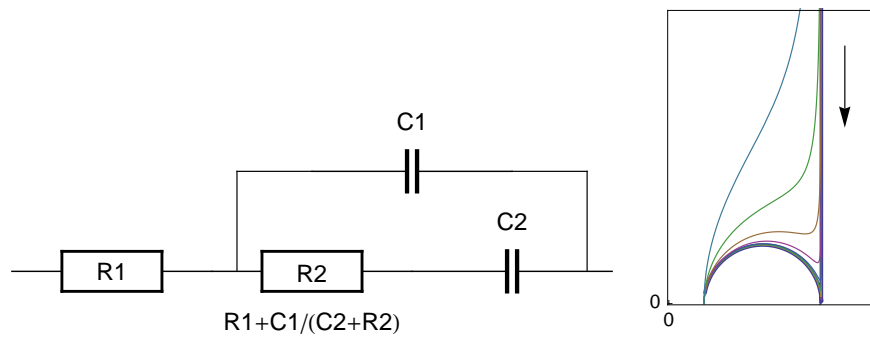


Figure 3.3:  $R_1+C_1/(C_2+R_2)$  circuit and Nyquist impedance diagrams.

**3.1.4  $R1+C1+R2/C2$  circuit**

Fig. 3.4.

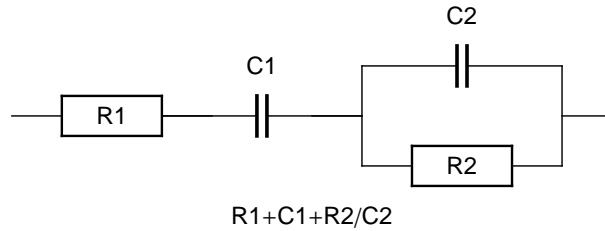


Figure 3.4:  $R1+C1+R2/C2$  circuit.

**3.1.5  $C1/(Q2+R2)$  circuit**

Fig. 3.5.

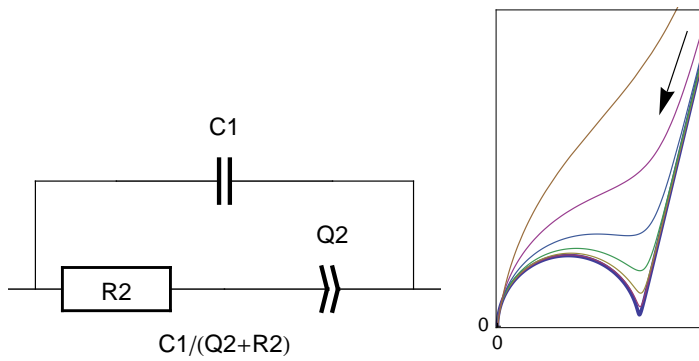


Figure 3.5:  $C1/(Q2+R2)$  circuit.

**3.1.6  $Q1+R2/C2$  circuit**

Fig. 3.6.

**3.1.7  $R1+C1/(Q2+R2)$  circuit**

Fig. 3.7.

**3.1.8  $R1+Q1+R2/C2$  circuit**

Fig. 3.8.

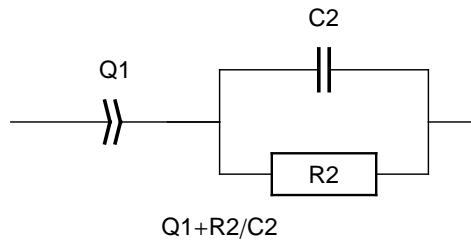


Figure 3.6:  $Q1+R2/C2$  circuit and Nyquist impedance diagrams.

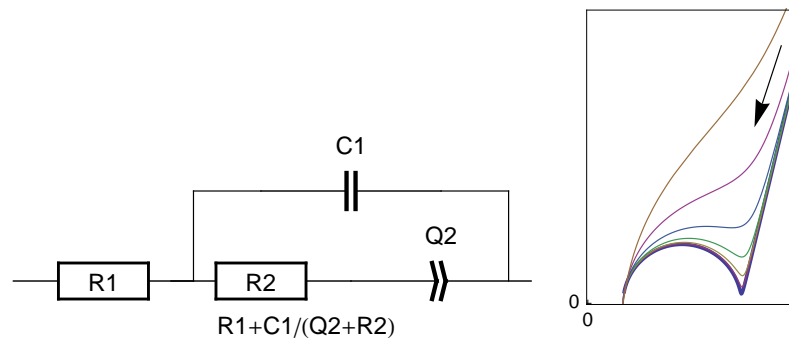


Figure 3.7:  $R1+C1/(Q2+R2)$  circuit and Nyquist impedance diagrams.

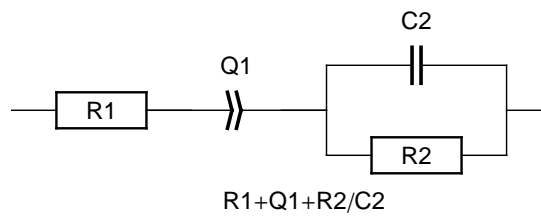


Figure 3.8:  $R1+Q1+R2/C2$  circuit.

### 3.1.9 $C1/(R2+W2)$ circuit

Fig. 3.9.

### 3.1.10 $R1+C1/(R2+W2)$ circuit

Fig. 3.10.

### 3.1.11 $W1+R2/C2$ circuit

Fig. 3.11.

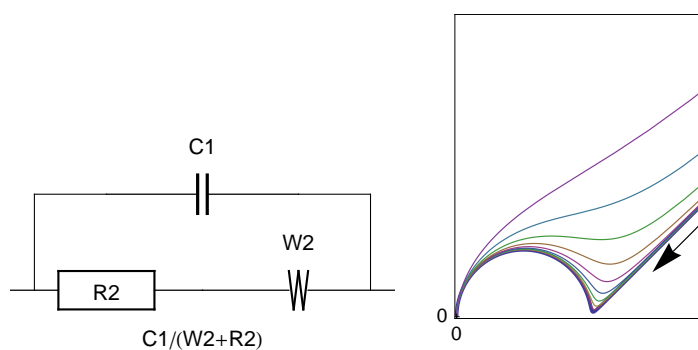


Figure 3.9:  $C1/(R2+W2)$  circuit (Randles circuit) and Nyquist impedance diagrams. (cf. Handbook of Electrochemical Impedance Spectroscopy. Diffusion impedances: <http://www.bio-logic.info/potentiostat/notesheis.html>)

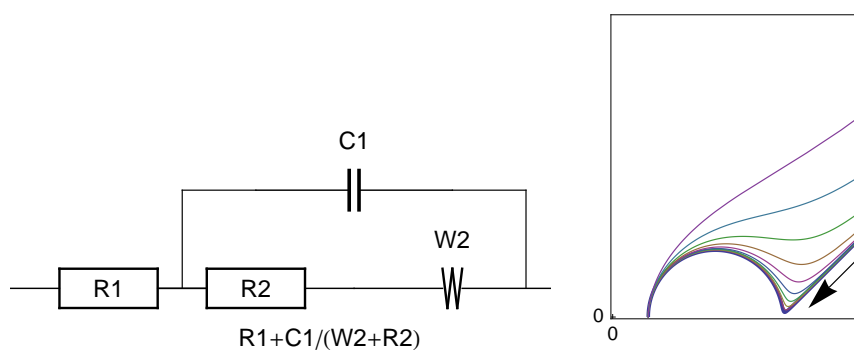


Figure 3.10:  $R1+C1/(R2+W2)$  circuit and Nyquist impedance diagrams.

### 3.1.12 $R1+W1+R2/C2$ circuit

Fig. 3.12.

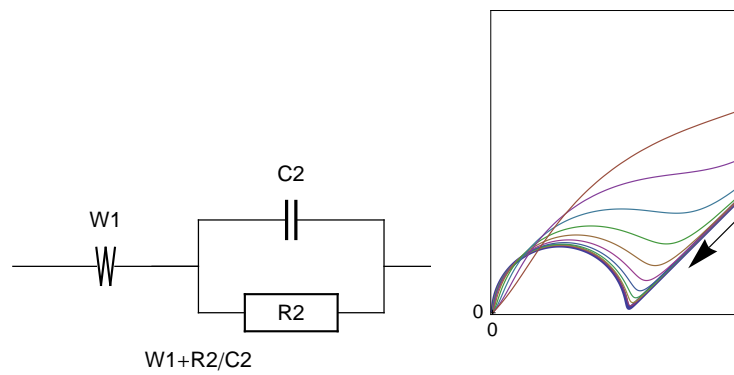


Figure 3.11:  $W_1 + R_2/C_2$  and Nyquist impedance diagrams.

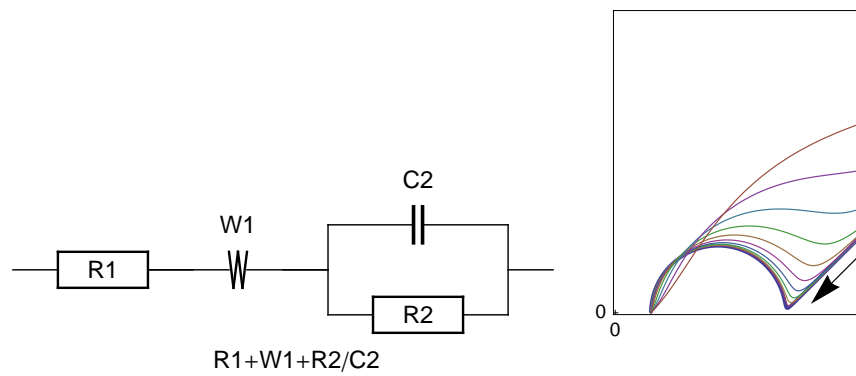
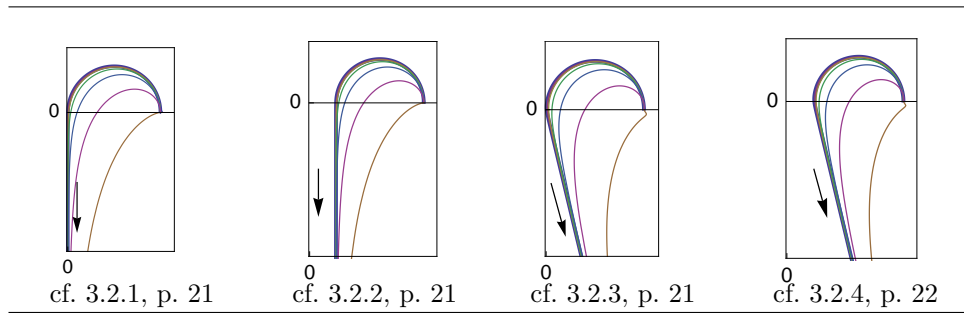


Figure 3.12:  $R_1 + W_1 + R_2/C_2$  circuit and Nyquist impedance diagrams.

### 3.2 Capacitive semicircle and inductive straight line

Table 3.2: Four different Nyquist diagrams made of one capacitive semicircle and one inductive straight line (limiting cases).



#### 3.2.1 $L1+R2/C2$ circuit

Fig. 3.13.

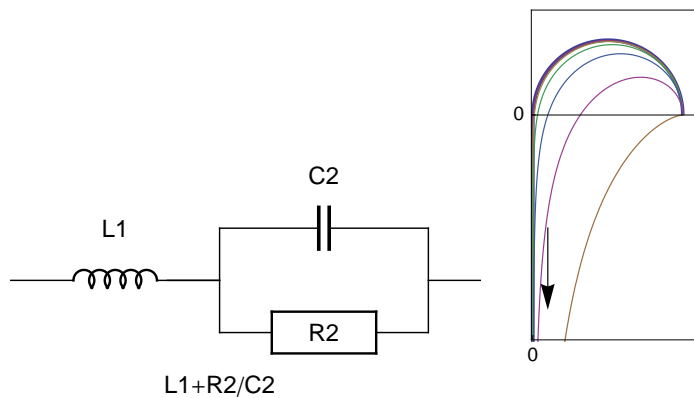


Figure 3.13:  $L1+R2/C2$  circuit and Nyquist impedance diagrams.

#### 3.2.2 $R1+L1+R2/C2$ circuit

Fig. 3.14.

#### 3.2.3 $Q1+R2/C2$ circuit, $\alpha1 < 0$

Fig. 3.15.

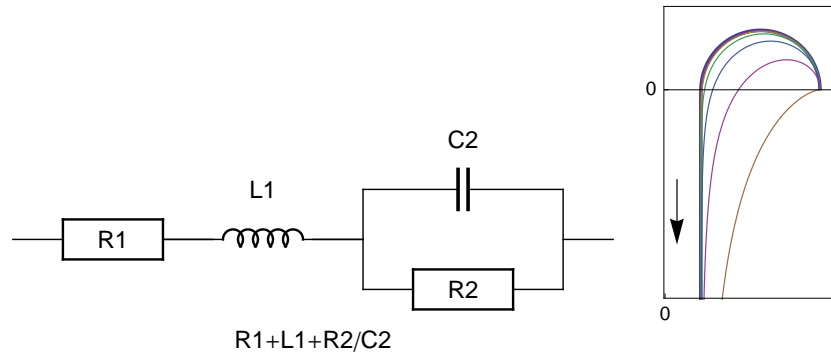


Figure 3.14:  $R_1+L_1+R_2/C_2$  circuit and Nyquist impedance diagrams.

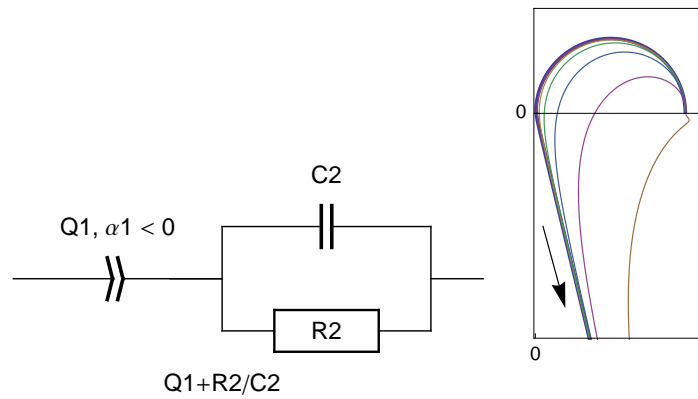


Figure 3.15:  $Q_1+R_2/C_2$  circuit.  $\alpha_1 < 0$ .

### 3.2.4 $R_1+Q_1+R_2/C_2$ circuit, $\alpha_1 < 0$

Fig. 3.16.

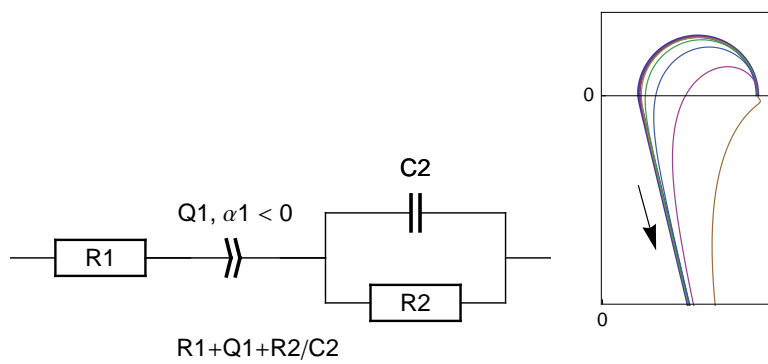
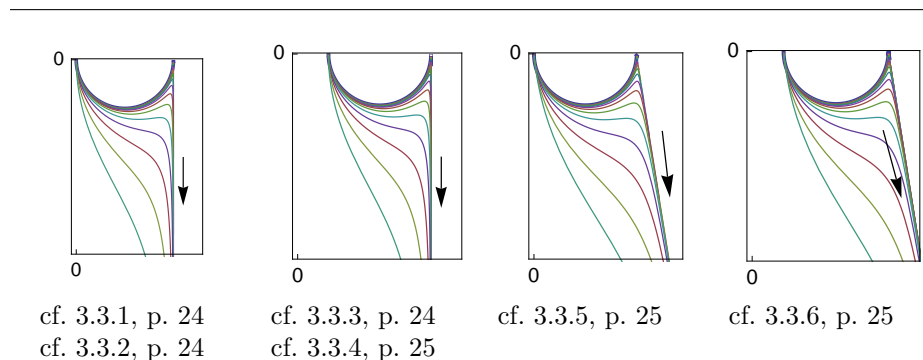


Figure 3.16:  $R_1 + Q_1 + R_2/C_2$  circuit and Nyquist impedance diagrams.  $\alpha_1 < 0$ .

### 3.3 Inductive semicircle and straight line

Table 3.3: Four different Nyquist diagrams made of one inductive semicircle and one inductive straight line (limiting cases).



#### 3.3.1 $R1/L1+L2$ circuit

Fig. 3.17.

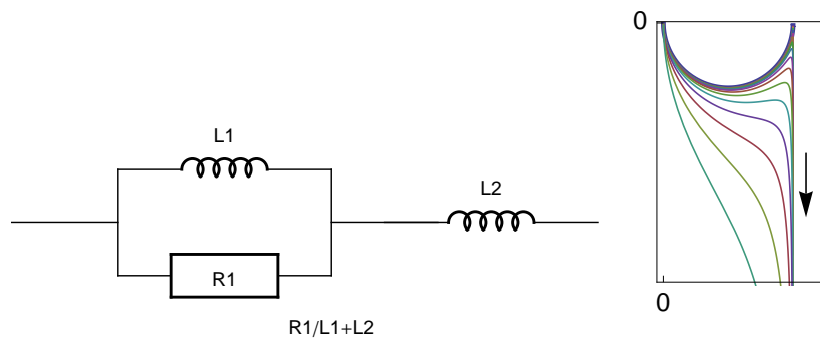


Figure 3.17:  $R1/L1+L2$  circuit and Nyquist impedance diagrams.

#### 3.3.2 $L1/(R1+L2)$ circuit

Fig. 3.18 <sup>(2)</sup>.

#### 3.3.3 $R1+R2/L2+L3$ circuit

Fig. 3.19.

<sup>2</sup> The  $R1/L1+L2$  and  $L1/(R1+L2)$  circuits are non-distinguishable, cf. Handbook of Electrochemical Impedance Spectroscopy. Circuits made of resistors and inductors: <http://www.bio-logic.info/potentiostat/notesheis.html>

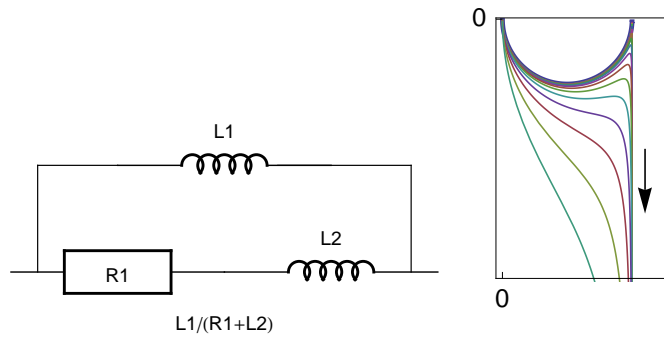


Figure 3.18:  $L1/(R1+L2)$  circuit and Nyquist impedance diagrams.

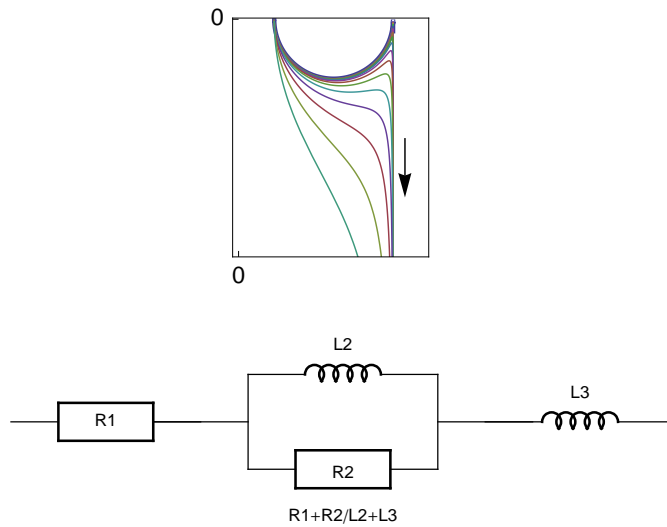


Figure 3.19:  $R1+R2/L2+L3$  circuit and Nyquist impedance diagrams.

### 3.3.4 $R1+L2/(R2+L3)$ circuit

Fig. 3.20.

### 3.3.5 $R1/L1+Q1$ circuit, $\alpha1 < 0$

Fig. 3.21.

### 3.3.6 $R1+R2/L2+Q1$ circuit, $\alpha1 < 0$

Fig. 3.22.

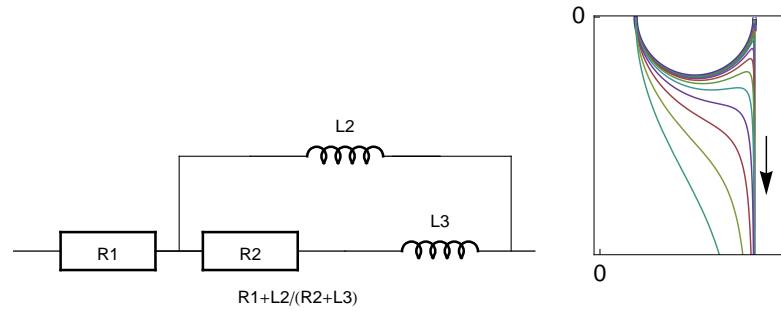


Figure 3.20:  $R_1 + L_2 / (R_2 + L_3)$  circuit and Nyquist impedance diagrams.

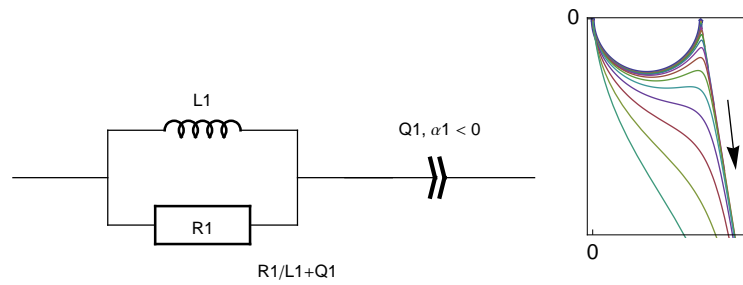


Figure 3.21:  $R_1 / L_1 + Q_1$  circuit ( $\alpha_1 < 0$ ) and Nyquist impedance diagrams.

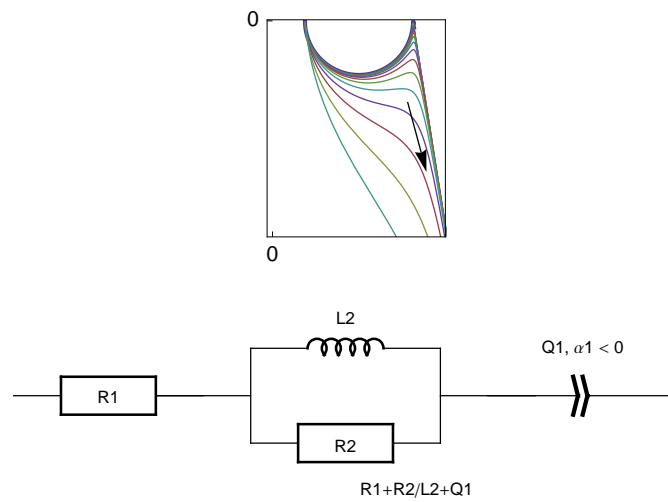
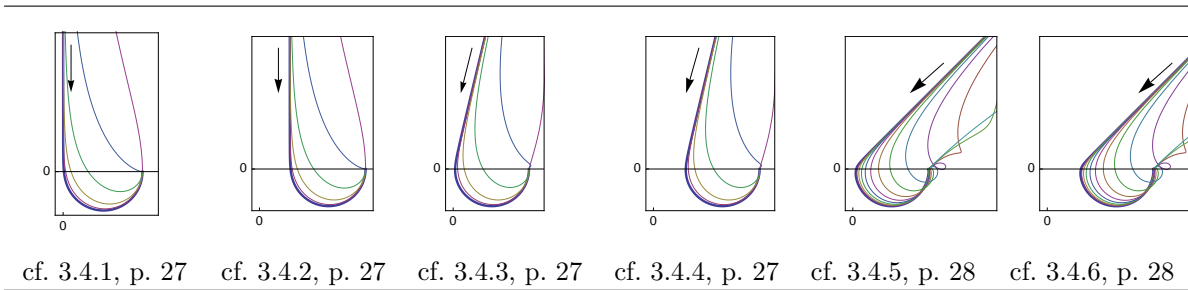


Figure 3.22:  $R_1 + R_2 / L_2 + Q_1$  circuit ( $\alpha_1 < 0$ ) and Nyquist impedance diagrams.

### 3.4 Inductive semicircle and capacitive straight line

Table 3.4: Six different Nyquist diagrams made of one inductive semicircle and one capacitive straight line.



#### 3.4.1 $R1/L1+C1$ circuit

Fig. 3.23.

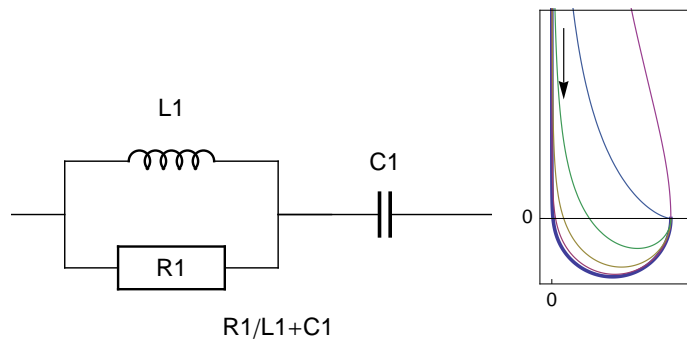


Figure 3.23:  $R1/L1+C1$  and Nyquist impedance diagrams.

#### 3.4.2 $R1+R2/L2+C1$ circuit

Fig. 3.24.

#### 3.4.3 $R1/L1+Q1$ circuit

Fig. 3.25.

#### 3.4.4 $R1+R2/L2+Q1$ circuit

Fig. 3.26.

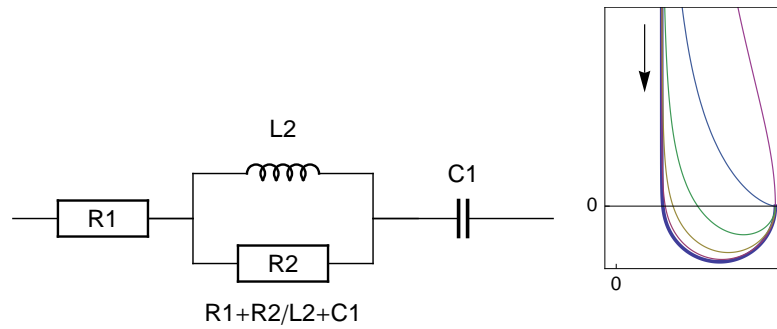


Figure 3.24:  $R1+R2/L2+C1$  and Nyquist impedance diagrams.

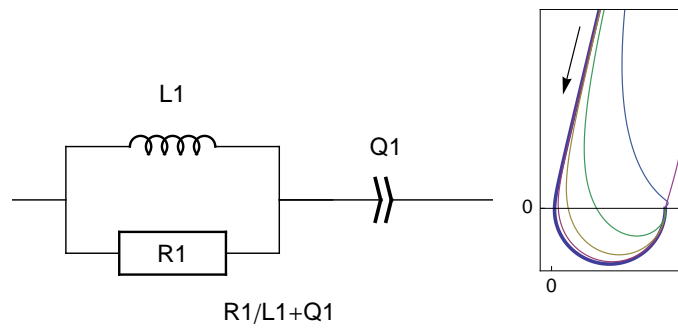


Figure 3.25:  $R1/L1+Q1$  circuit and Nyquist impedance diagrams.

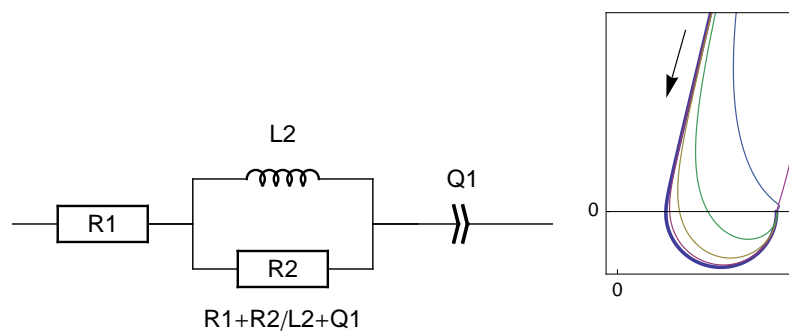


Figure 3.26:  $R1+R2/L2+Q1$ .

### 3.4.5 $R1/L1+W1$ circuit

Fig. 3.27.

### 3.4.6 $R1+R2/L2+W1$ circuit

Fig. 3.28.

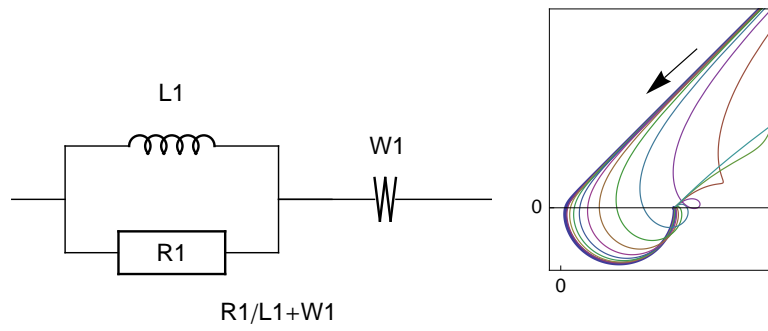


Figure 3.27:  $R1/L1+W1$  circuit and Nyquist impedance diagrams.

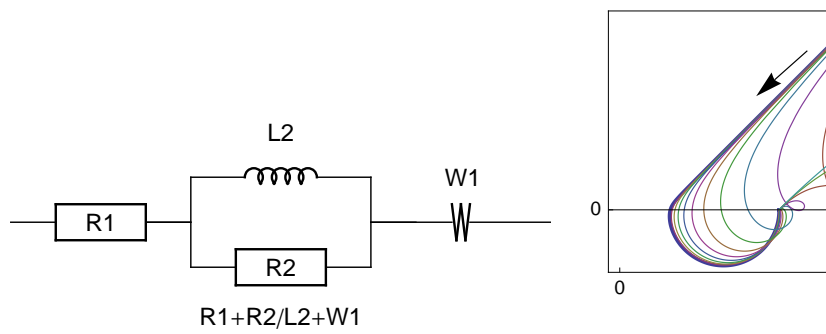


Figure 3.28:  $R1+R2/L2+W1$  and Nyquist impedance diagrams.



# Bibliography

- [1] BRUG, G. J., VAN DEN EEDEN, A. L. G., SLUYTERS-REHBACH, M., AND SLUYTERS, J. H. The analysis of electrode impedance complicated by the presence of a constant phase element. *J. Electroanal. Chem.* 176 (1984), 275–295.
- [2] LONNÉ, Q., GLANDUT, N., LABBE, J.-C., AND LEFORT, P. Fabrication and characterization of ZrB<sub>2</sub>-SiC ceramic electrodes coated with a proton conducting, SiO<sub>2</sub>-rich glass layer. *Electrochimica Acta In Press, Corrected Proof* (2011), –.