How to fit transmission lines with ZFit

I Introduction

ZFit is the impedance fitting tool of EC-Lab®. This note will describe how to fit transmission lines using one equivalent circuit elements contained in ZFit.
It is well known that the Warburg impedance is equivalent to that of a semi-infinite large network *i.e.* a transmission line, as shown in Fig. 1 [1, 2]. Moreover, the transmission lines are often used for modeling porous electrodes, for example in the field of photovoltaics.

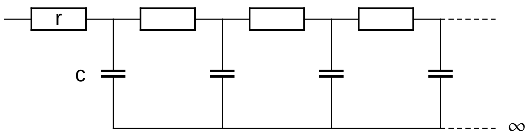

Fig. 1: The equivalent circuit of the Warburg impedance.

More recently it has been shown [3] that the impedance of a L-long transmission line made of χ and ζ elements and terminated by a Z_{L} element (Fig. 2) is given by the general expression:

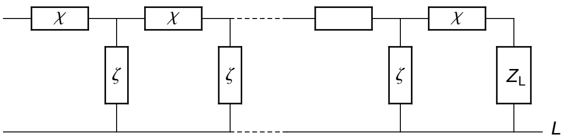

Fig. 2: Uniform transmission line made of χ and ζ elements and terminated by Z_{L} [3].

$$Z = \frac{(\zeta\,\chi - Z_{\mathrm{L}}^2)\,\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)}{Z_{\mathrm{L}}\,\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) + \sqrt{\zeta\,\chi}\,\mathrm{ch}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)} + Z_{\mathrm{L}}$$

with three limiting cases
- open-circuited transmission line

$$Z_{\mathrm{L}} = \infty \Rightarrow Z = \sqrt{\zeta\,\chi}\,\mathrm{coth}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \quad (1)$$

- short-circuited transmission line

$$Z_{\mathrm{L}} = 0 \Rightarrow Z = \sqrt{\zeta\,\chi}\,\mathrm{th}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \quad (2)$$

- semi-infinite transmission line

$$L \to \infty \Rightarrow Z = \sqrt{\zeta\,\chi} \quad (3)$$

Hereafter, some transmission lines are described and the corresponding "simple" equivalent circuit elements are shown. The open-circuited transmission lines will be explained, followed by short-circuited and semi-infinite transmission lines.

II Open-circuited transmission lines $Z_{\mathrm{L}} = \infty$

II.1 Open-circuited URC (Uniform distributed RC)

Let us consider the open-circuited tranmission line made of r and c elements (Fig. 3).

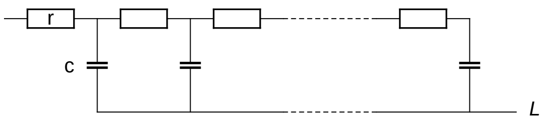

Fig. 3: L-long open uniform distributed RC (URC) transmission line [4, 5].

Using Eq. (1), the impedance of the URC transmission line is given by ([1])

$$\chi = r,\ \zeta = \frac{1}{j\,\omega\,c} \Rightarrow Z = \sqrt{r}\,\frac{\mathrm{coth}(L\sqrt{r\,c\,j\,\omega})}{\sqrt{c\,j\,\omega}}$$

[1]The transmission lines are named according to the U-$\chi\zeta$ format where U means uniform distributed and χ and ζ are the element of the transmission line.

with $\omega = 2\pi f$. This impedance is similar to that of the M element of ZFit

$$Z_{\mathrm{M}} = R_{\mathrm{d}} \frac{\coth\sqrt{\tau_{\mathrm{d}}\, j\,\omega}}{\sqrt{\tau_{\mathrm{d}}\, j\,\omega}}, R_{\mathrm{d}} = L\, r,\ \tau_{\mathrm{d}} = L^2\, r\, c$$

II.2 Open-circuited URQ

Replacing c elements by q elements, with $Z_{\mathrm{q}} = 1/(q\,(j\,\omega)^\alpha)$, leads to transmisssion line shown in Fig. 4.

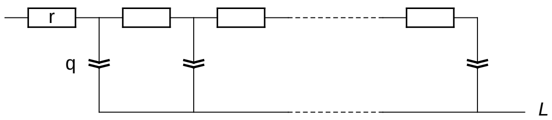

Fig. 4: L-long open uniform distributed RQ (URQ) transmission line.

The transmission line impedance is given by

$$\chi = r,\ \zeta = \frac{1}{q\,(j\,\omega)^\alpha} \Rightarrow$$
$$Z = \sqrt{r}\,\frac{\coth(L\,\sqrt{r\,q}\,(j\,\omega)^{\alpha/2})}{\sqrt{q}\,(j\,\omega)^{\alpha/2}}$$

This impedance is similar to that of the $\mathrm{M_a}$ element of ZFit

$$Z_{\mathrm{M_a}} = R\,\frac{\coth(\tau\, j\,\omega)^{\alpha/2}}{(\tau\, j\,\omega)^{\alpha/2}} \text{ with}$$
$$R = L\, r,\ \tau = (L^2\, r\, q)^{1/\alpha}$$

As an example a Nyquist impedance diagram of a battery Ni-MH 1900 mAh is shown in Fig. 5. The equivalent circuit R1+L1+Q1/(R2+Ma3), containing a $\mathrm{M_a}$ element, is chosen to fit the data shown in Fig. 5. The values of the parameters, obtained using the ZFit tool of EC-Lab, are $R1 = 0.049\ \Omega$, $L1 = 0.154 \times 10^{-6}$ H, $Q1 = 0.66$ F s$^{\alpha-1}$, $\alpha 1 = 0.61$, $R2 = 0.0236\ \Omega$, $R3 = L\, r = 0.057\,\Omega$, $\tau 3 = (L^2\, r\, q)^{1/\alpha} = 2.25$ s and $\alpha 3 = 0.89$.

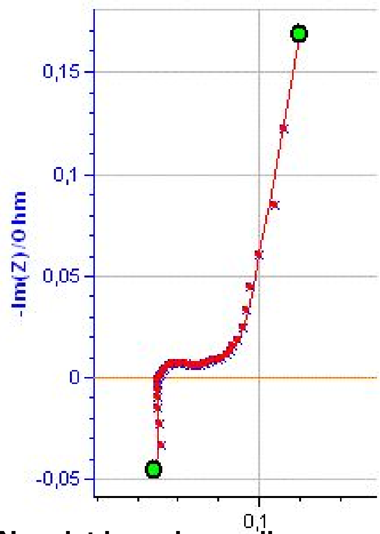

Fig. 5: Nyquist impedance diagram of a battery Ni-MH 1900 mAh.

II.3 Open-circuited UQC

The equivalent circuit of the so-called anomalous diffusion is show in Fig. 6 [6].

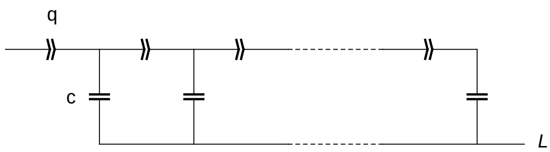

Fig. 6: L-long open uniform distributed QC (UQC) transmission line. Anomalous diffusion [6].

The anomalous diffusion impedance is given by

$$\chi = \frac{1}{q\,(j\,\omega)^\alpha},\ \zeta = \frac{1}{c\, j\,\omega} \Rightarrow$$
$$Z = \frac{\coth\left(L\,\sqrt{\frac{c}{q}}\,(j\,\omega)^{\frac{1}{2}-\frac{\alpha}{2}}\right)}{\sqrt{c\,q}\,(j\,\omega)^{\frac{\alpha}{2}+\frac{1}{2}}}$$

This impedance is similar to that of the $\mathrm{M_g}$ element of ZFit

$$Z_{\mathrm{M_g}} = R\,\frac{\coth(\tau\, j\,\omega)^{\gamma/2}}{(\tau\, j\,\omega)^{1-\gamma/2}} \text{ with } \gamma = 1-\alpha,$$
$$R = c^{\frac{1}{\gamma}-1} L^{\frac{2}{\gamma}-1} q^{-1/\gamma},\ \tau = c^{\frac{1}{\gamma}} L^{2/\gamma} q^{-1/\gamma}$$

III Short-circuited transmission lines $Z_L = 0$

III.1 Short-circuited URC

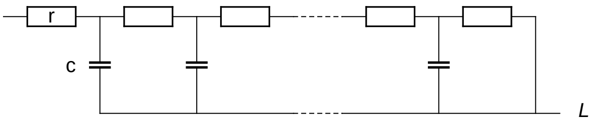

Fig. 7: L-long short-circuited uniform distributed RC (URC) transmission line.

Using Eq. (2), the impedance of the short-circuited transmission line made of r and c elements (Fig. 7) is given by

$$\chi = r, \zeta = \frac{1}{c\,j\,\omega} \Rightarrow$$

$$Z = r\frac{\text{th}\,(L\sqrt{r\,c\,j\,\omega})}{\sqrt{r\,c\,j\,\omega}} \quad (4)$$

This impedance is similar to that of the W_d element of ZFit

$$Z_{W_d} = R_d\frac{\text{th}\sqrt{\tau_d\,j\,\omega}}{\sqrt{\tau_d\,j\,\omega}},\ R_d = L\,r, \tau_d = L^2\,r\,c$$

IV Semi-infinite transmission lines $L \to \infty$

IV.1 Semi-infinite URC

The impedance of the semi-infinite transmission line shown in Fig. 1 is obtained making $L \to \infty$ in Eq. (4).

$$L \to \infty \Rightarrow Z = r\frac{\text{th}\,(L\sqrt{r\,c\,j\,\omega})}{\sqrt{r\,c\,j\,\omega}} \approx \frac{\sqrt{r}}{\sqrt{c\,j\,\omega}}$$

This expression is similar to that of the Warburg (W) element of ZFit

$$Z_W = \frac{2\,\sigma}{\sqrt{j\,\omega}} \text{ with } \sigma = \frac{\sqrt{r}}{2\sqrt{c}}$$

As an example a Nyquist impedance diagram of a Fe(II)/Fe(III) system is shown in Fig. 8.

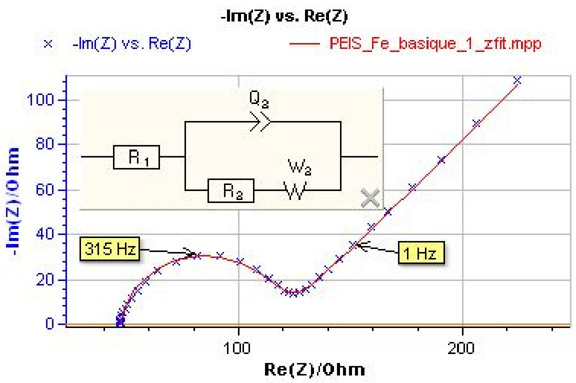

Fig. 8: Nyquist impedance diagram of a Fe(II)/Fe(III) system in basic medium.

The Randles circuit R1+Q2/(R2+W2), containing a Warburg element, is chosen to fit the data shown in Fig. 8. The values of the parameters for equivalent circuit are $R_1 = 47.57\ \Omega$, $Q_2 = 17.09 \times 10^{-6}$ F $s^{\alpha-1}$, $\alpha = 0.885$, $R_2 = 70.94\ \Omega$ and $\sigma_2 = 85.33\ \Omega$ $s^{-1/2} \Rightarrow \sqrt{r/c} = 42.7$ Ω $s^{-1/2}$.

IV.2 Semi-infinite URRC

First of all, let us calculate the impedance of the L-long URRC transmission line (Fig. 9) corresponding to diffusion-reaction and diffusion-trapping impedance [7]:

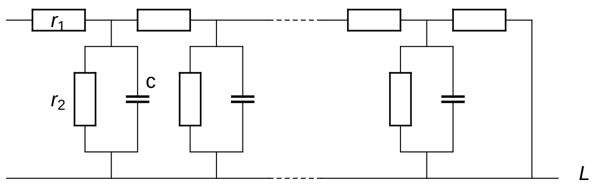

Fig. 9: L-long short-circuited uniform distributed RRC (URRC) transmission line.

$$\chi = r_1,\ \zeta = \frac{r_2}{1 + r_2\,c\,j\,\omega} \Rightarrow$$

$$Z = \sqrt{r_1\, r_2}\,\frac{\text{th}\left(L\sqrt{\frac{r_1}{r_2}(1+r_2\, c\, j\, \omega)}\right)}{\sqrt{1+r_2\, c\, j\, \omega}}$$

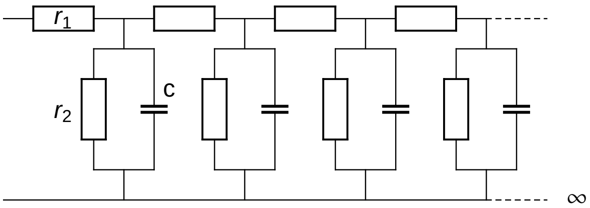

Fig. 10: Semi-infinite short-circuited uniform distributed RRC (URRC) transmission line.

With $L \to \infty$ it is obtained [8]:

$$L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1\, r_2}}{\sqrt{1+r_2\, c\, j\, \omega}}$$

This expression is similar to that of the Gerischer element G of ZFit [9]:

$$Z_{\text{G}} = \frac{R_{\text{G}}}{\sqrt{1+\tau_{\text{G}}\, j\, \omega}},\ R_{\text{G}} = \sqrt{r_1\, r_2},\ \tau_{\text{G}} = r_2\, c$$

IV.3 Semi-infinite URRQ

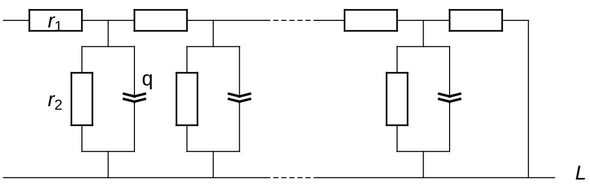

Fig. 11: L-long short-circuited uniform distributed RRQ (URRQ) transmission line.

Replacing c elements by q elements

$$\chi = r_1,\ \zeta = \frac{r_2}{1+r_2\, q\, (j\, \omega)^{\alpha}} \Rightarrow$$

$$Z = \sqrt{r_1\, r_2}\,\frac{\text{th}\left(L\sqrt{\frac{r_1}{r_2}(1+r_2\, q\, (j\, \omega)^{\alpha})}\right)}{\sqrt{1+r_2\, q\, (j\, \omega)^{\alpha}}}$$

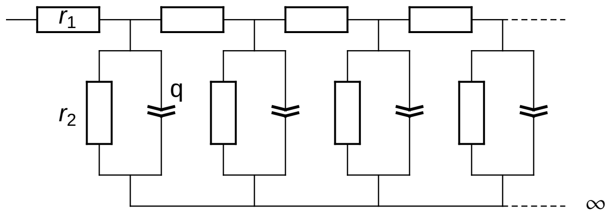

Fig. 12: Semi-infinite short-circuited uniform distributed RRQ (URRQ) transmission line.

and

$$L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1\, r_2}}{\sqrt{1+r_2\, q\, (j\, \omega)^{\alpha}}}$$

This expression is similar to that of the G_{a} element of ZFit

$$Z_{\text{G}_\text{a}} = \frac{R}{\sqrt{1+\tau\, (j\, \omega)^{\alpha}}},\ R = \sqrt{r_1\, r_2},\ \tau = r_2\, q$$

V Conclusion

Seven elements, W, Wd, M, Ma, Mg, G and Ga, available in ZFit, can be used to represent the impedance of seven different transmission lines, as summarized in the table below (Tabs. 1 (p. 4), 2 (p. 6)).

Table 1: Summary table.

Transmission line		ZFit Element
Open Circuited	URC	M
	URQ	Ma
	UQC	Mg
Short circuited	URC	Wd
Semi-∞	URC	W
	URRC	G
	URRQ	Ga

References

[1] J. C. WANG, *J. Electrochem. Soc.* **134**, 1915 (1987).

[2] M. SLUYTERS-REHBACH, *Pure & Appl. Chem.* **66**, 1831 (1994).

[3] J. BISQUERT, *Phys. Chem. Chem. Phys.* **2**, 4185 (2000).

[4] G. C. TEMES and J. W. LAPATRA, *Introduction to Circuits Synthesis and Design*, McGraw-Hill, New-York, 1977.

[5] J.-P. DIARD, B. LE GORREC, and C. MONTELLA, *J. Electroanal. Chem.* **471**, 126 (1999).

[6] J. BISQUERT and A. COMPTE, *J. Electroanal. Chem.* **499**, 112 (2001).

[7] J.-P. DIARD and C. MONTELLA, *J. Electroanal. Chem.* **557**, 19 (2003).

[8] B. A. BOUKAMP and H. J.-M. BOUWMEESTER, *Solid State Ionics* **157**, 29 (2003).

[9] H. GERISCHER, *Z. Physik. Chem. (Leipzig)* **198**, 286 (1951).

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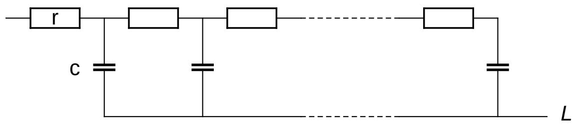
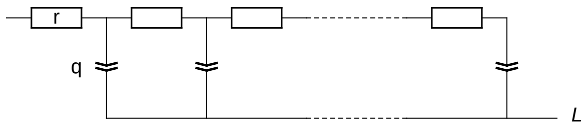
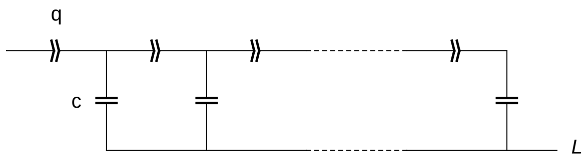
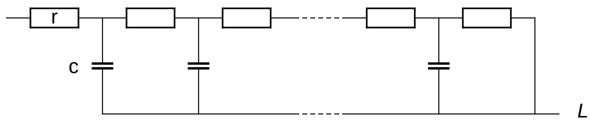
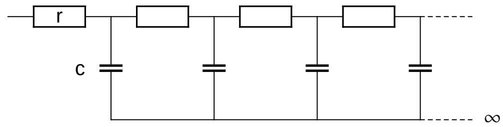
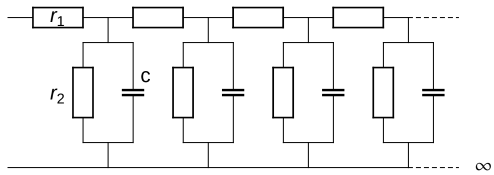
Table 2: ZFit elements vs. transmission lines.

ZFit element	Equations	Transmission line
M	$R_\mathrm{d}\dfrac{\coth\sqrt{\tau_\mathrm{d}\,j\,\omega}}{\sqrt{\tau_\mathrm{d}\,j\,\omega}}$ $R_\mathrm{d}=L\,r,\ \ \tau_\mathrm{d}=L^2\,r\,c$	
M_a	$R\dfrac{\coth(\tau\,j\,\omega)^{\alpha/2}}{(\tau\,j\,\omega)^{\alpha/2}}$ $R=L\,r$ $\tau=(L^2\,r\,q)^{1/\alpha}$	
M_g	$R\dfrac{\coth(\tau\,j\,\omega)^{\gamma/2}}{(\tau\,j\,\omega)^{1-\gamma/2}}$ $R=c^{\frac{1}{\gamma}-1}L^{\frac{2}{\gamma}-1}q^{-1/\gamma}$ $\tau=c^{\frac{1}{\gamma}}L^{2/\gamma}q^{-1/\gamma}$	
W_d	$R_\mathrm{d}\dfrac{\mathrm{th}\sqrt{\tau_\mathrm{d}\,j\,\omega}}{\sqrt{\tau_\mathrm{d}\,j\,\omega}}$ $R_\mathrm{d}=L\,r$ $\tau_\mathrm{d}=L^2\,r\,c$	
W	$\dfrac{2\,\sigma}{\sqrt{j\,\omega}}$ $\sigma=\dfrac{\sqrt{r}}{2\sqrt{c}}$	
G	$\dfrac{R_\mathrm{G}}{\sqrt{1+\tau_\mathrm{G}\,j\,\omega}}$ $R_\mathrm{G}=\sqrt{r_1\,r_2}$ $\tau_\mathrm{G}=r_2\,c$	
G_a	$\dfrac{R_\mathrm{G}}{\sqrt{1+\tau_\mathrm{G}\,(j\,\omega)^\alpha}}$ $R_\mathrm{G}=\sqrt{r_1\,r_2}$ $\tau_\mathrm{G}=r_2\,q$